

Mini-groupwork 1, solutions, 2014

a)

$$h = \frac{42 \text{ mm}}{7 \text{ mm}} \cdot 6378 \text{ km} \cdot 2 = 77000 \text{ km}$$

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 1.5 \cdot 10^6}{2} = 1 \cdot 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 4.7 \cdot 10^6 \text{ ms}^{-1}$$

Approximating the loop with a quarter-circle, the electron has to travel a length $s = \pi h/2 = 120\,000 \text{ km}$

Then we get $t = 25 \text{ s}$.

b)

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m} \Rightarrow$$

$$B = \frac{2\pi f_c m}{q} = \frac{2\pi \cdot 1 \cdot 10^{10} \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.36 \text{ T}$$

The perpendicular energy is given by

$$E = 2 \cdot \frac{k_B T}{2} = 2 \cdot \frac{1.38 \cdot 10^{-23} \cdot 1.5 \cdot 10^6}{2} = 2 \cdot 10^{-17} \text{ J}$$

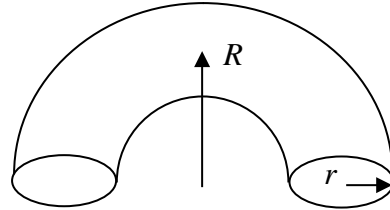
\Rightarrow

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 6.6 \cdot 10^6 \text{ ms}^{-1}$$

$$\rho = \frac{m_e v_{\perp}}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 6.6 \cdot 10^6}{1.6 \cdot 10^{-19} \cdot 0.36} = 1.0 \cdot 10^{-4} \text{ m}$$

c)

Model the flare by a half torus with minor axis r , and major axis R . From the figure I estimate $R = 2.6 R_E$, and $r = 2 R_E$.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.36$ T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

$$W = V p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 2 \cdot 12^2 R_E^3 \frac{B^2}{2\mu_0} = \pi^2 \cdot 2 \cdot 12^2 (6378 \cdot 10^3)^3 \frac{(0.36)^2}{2\mu_0} \\ = 3.8 \cdot 10^{28} \text{ J}$$

which is enough to produce a solar flare, even if only a few per cent of the available magnetic field energy is converted into kinetic energy of the plasma.